Lateral and Drag Forces on Misaligned Cylindrical Rollers

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Formulas are presented for estimating the drag and lateral forces on misaligned cylindrical rollers.

I. Introduction

In DSN Progress Report 42-45 (Ref. 1), a concept was presented which accounted for the maximum lateral forces induced by a misaligned steel cylindrical wheel rolling on a flat track. The concept may be understood by referring to Fig. 1, which depicts the displacement as being comprised of two components, namely, a frictionless rolling perpendicular to the axis of the roller followed by a sliding parallel to the roller axis. The concept is that the two actions occur simultaneously. Since the rolling action is assumed to be frictionless, the resultant frictional force acting on the roller is directed along the roller axis in the sense indicated in Fig. 1. The components of the resultant frictional force $W\mu$ acting on the roller along the x and y axes are:

$$F_x = -W\mu \sin \alpha \quad (1)$$

$$F_y = W\mu \cos \alpha \quad (2)$$

where W is the normal force between the roller and track and μ is the effective coefficient of sliding friction.

Although the foregoing concept produces a lateral force which is approximately the same as the maximum values obtained from tests, it is not valid for very small values of the misalignment angle α. For example, Eq. 2 indicates that the maximum lateral force occurs when the misalignment angle is zero, whereas tests show that the lateral force is zero for zero misalignment, but increases to the full value of $W\mu$ within one degree of misalignment.

The present report is a description of a concept which yields formulas for the lateral force and the drag force in terms of the misalignment angle α, the coefficient of sliding friction μ, and the rolling friction coefficient f, which pertains to the condition of zero misalignment. From tests on various roller bearings, good estimates of f can be obtained. Values of the sliding coefficient of friction μ can be obtained from many sources.

II. Description of the Concept

The drag resistance of a roller comes from several independent sources, among which are material hysteresis, viscosity of lubricant, and sliding friction caused by a variation in the rolling radius. The last effect, for the case of a cylindrical roller on a track of greater width, may be understood by realizing that the contact area between the loaded roller and track is not flat but curved, such that the edge of the roller has a smaller rolling radius than that of the center of the roller. The total drag coefficient for a perfectly aligned roller f is some combination of these factors, and its range and values can be obtained from numerous published reports.

The concept used here is that the drag resistance of an aligned roller is caused only by differential sliding of two different roller radii. The sliding on the smaller radius is opposite to the roller motion and the sliding on the larger radius is in the direction of the roller motion, and the effect of their difference is equal to Wf. If the roller undergoes an angular misalignment α, a sliding along the roller axis occurs. The resultant sliding direction is obtained by geometrically adding the two sliding components, thus obtaining one direction for the small radius and another direction for the large radius. The resultant frictional forces must be coincident with these resultant sliding directions. The total frictional force acting on the roller is assumed to be the vector sum of the resultant from the small radius and the resultant from the large radius. The side force F_S and the drag force F_D are components of this total frictional force.

In the following analysis of the forces it is assumed that the bearing between the roller and its axle is frictionless.

III. Analysis of the Forces

Figure 2 shows two positions of the wheel or roller. The wheel is composed of radii r and R where $r < R$. Let the mean of these two radii be ρ and assume that this ficticious ρ rolls without sliding along the line α. As the wheel travels the distance U along the X axis, the r radius slips along line α by the amount $U[1 - (r/\rho)]\cos\alpha$ in the sense shown in the figure. From the figure it is clear that the radius r also slips perpendicularly to line α by the amount $U \sin\alpha$ in the sense indicated in the figure. Similarly radius R slips along line α by the amount $U[(R/\rho) - 1]\cos\alpha$ and slips perpendicularly to line α by the amount $U \sin\alpha$ in the senses shown in Fig. 2. The sliding frictional forces associated with radii r and R, namely, $Wa\mu$ and $W(1-a)\mu$ respectively, where a is the fraction of the total normal force applied to radius r, will have the same directions as the resultant slip lines for radii r and R. Considering the wheel as a free body, the senses of the forces $Wa\mu$ and $W(1-a)\mu$ are as shown in Fig. 2.

By referring to Fig. 2, two equations of equilibrium may be written by summing forces along line α and by summing moments about the wheel center, as follows:

$$Wa\mu\cos\gamma - W(1-a)\mu\cos\beta = Wf \tag{3}$$

$$Wa\mu r\cos\gamma - W(1-a)\mu R\cos\beta = 0 \tag{4}$$

When α is zero both γ and β are zero. For this condition, Eqs. (3) and (4) become:

$$Wa\mu - W(1-a)\mu = Wf \tag{5}$$

$$Wa\mu r = W\mu(1-a)R \tag{6}$$

From (5)

$$a = (1 + f/\mu)/2 \tag{7}$$

It is assumed that this value of a holds for any value of α. By substituting (7) into (6) the following is obtained:

$$\frac{R}{r} = \frac{a}{1-a} = \frac{1+f/\mu}{1-f/\mu} \tag{8}$$

The radius ρ was defined as the mean of r and R; thus from (8),

$$\rho = \frac{r+R}{2} = \frac{r}{2}\left[1 + \frac{1+f/\mu}{1-f/\mu}\right]$$

$$= \frac{R}{2}\left[1 + \frac{1-f/\mu}{1+f/\mu}\right] \tag{9}$$

$$\rho = \frac{r}{1-f/\mu} = \frac{R}{1+f/\mu} \tag{10}$$

$$1 - \frac{r}{\rho} = \frac{R}{\rho} - 1 = f/\mu \tag{11}$$

Using the expressions for $1 - (r/\rho)$ and $(R/\rho) - 1$ as given by Eq. (11), namely f/μ, the angle γ and β of Fig. 2 may be evaluated thus:

$$\tan\gamma = \frac{U\sin\alpha}{U\left(\frac{f}{\mu}\right)\cos\alpha} = \frac{\mu}{f}\tan\alpha = \tan\beta \tag{12}$$

From (12) the following expressions for $\sin\gamma$ and $\cos\gamma$ may be obtained:

$$\sin\gamma = \frac{\mu\tan\alpha}{\sqrt{\mu^2\tan^2\alpha + f^2}} = \sin\beta \tag{13}$$

$$\cos\gamma = \frac{f}{\sqrt{\mu^2\tan^2\alpha + f^2}} = \cos\beta \tag{14}$$

The drag force F_D and the side force F_S are obtained respectively by summing the x and y components of the forces μWa and $\mu W(1-a)$, shown in Fig. 2, obtaining the following:

$$F_D = -[aW\mu \cos\gamma - (1-a)\, W\mu \cos\beta] \cos\alpha$$

$$-[aW\mu \sin\gamma + (1-a)\, W\mu \sin\beta] \sin\alpha \qquad (15)$$

$$F_S = [aW\mu \sin\gamma + (1-a)\, W\mu \sin\beta] \cos\alpha$$

$$+[(1-\alpha)\, W\mu \cos\beta - aW\mu \cos\gamma] \sin\alpha \qquad (16)$$

Substituting Eqs. (7) (13) and (14) into (15) and (16), there are obtained:

$$\frac{F_D}{W} = \frac{\mu^2 \tan\alpha \sin\alpha + f^2 \cos\alpha}{\sqrt{\mu^2 \tan^2\alpha + f^2}} \qquad (17)$$

$$\frac{F_S}{W} = \frac{\mu^2 \tan\alpha \cos\alpha - f^2 \sin\alpha}{\sqrt{\mu^2 \tan^2\alpha + f^2}} \qquad (18)$$

Equations (17) and (18) express the drag force and side force in terms of the sliding coefficient of friction μ and the aligned roller coefficient f for any value of the misalignment angle α.

Some of the test results from Ref. 1 are presented in Fig. 3. The solid line pertains to the conditions of a clean track, whereas the dashed line represents the average of all the tests made. Two sets of μ and f were evaluated by Eq. (18) and compared to the test results.

IV. Conclusion

A simple analysis has yielded a formula for the lateral force coefficient of a misaligned cylindrical roller on a flat track. Reasonable values for the sliding coefficient of friction μ and for the rolling drag coefficient f when inserted into the formula produce values which agree quite well with the lateral force test results of Ref. 2.

The drag force coefficient was calculated for a misalignment value of 13.7 arc minutes and compared with the measured values obtained on a model of an antenna wheel and track system. The agreement as reported in Ref. 3 was good, but very few tests were made.

A useful application of Eqs. (17) and (18) is the estimation of drag torque of an antenna employing a wheel and track bearing system, and the estimation of the lateral forces induced on the wheel by a slight misalignment.

The present analysis gives no explanation of the "stick-slip" behavior of certain misaligned rollers reported in Ref. 2, and observed on the radial bearing of the 64-meter antenna at DSS 14 when the roller misalignment exceeds a certain value. It is hoped that further study will bring forth a quantitative explanation of this phenomenon.

References

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3. McGinness, H., "Antenna Azimuth Bearing Model Experiment," *DSN Progress Report 42-53, July and August 1979*, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1979.

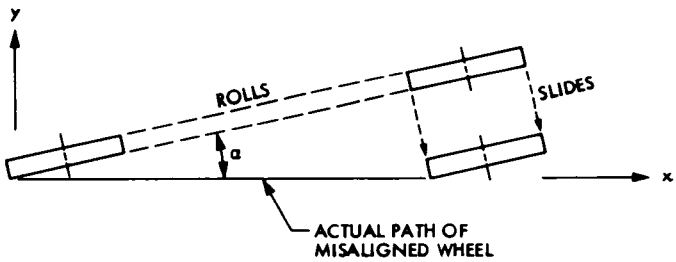

Fig. 1. Model of induced lateral force for frictionless wheel

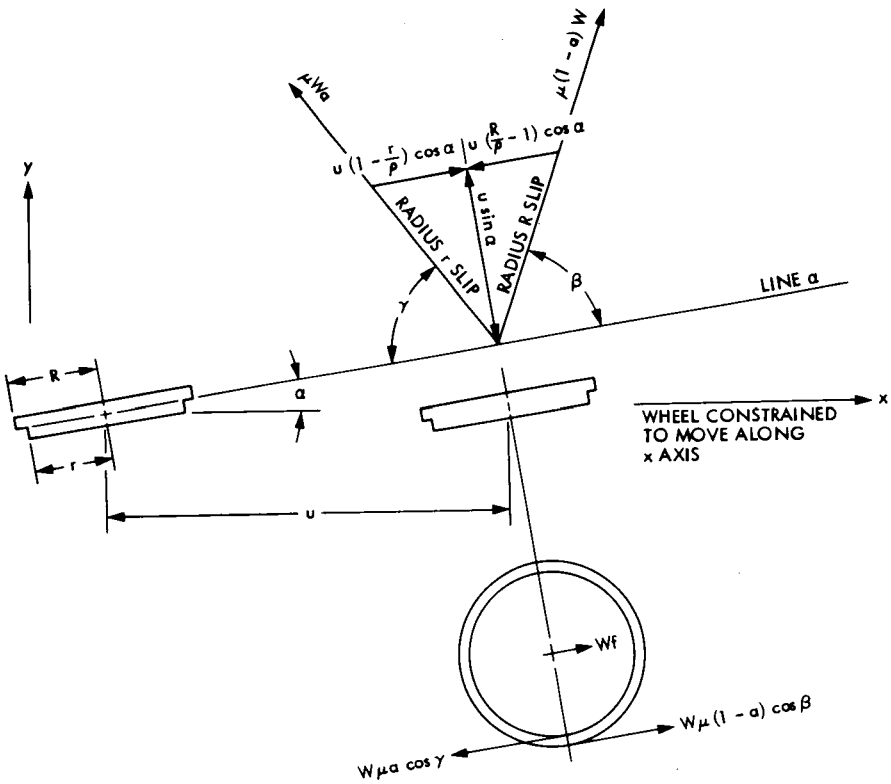

Fig. 2. Directions of slip of radius *r* and radius *R*

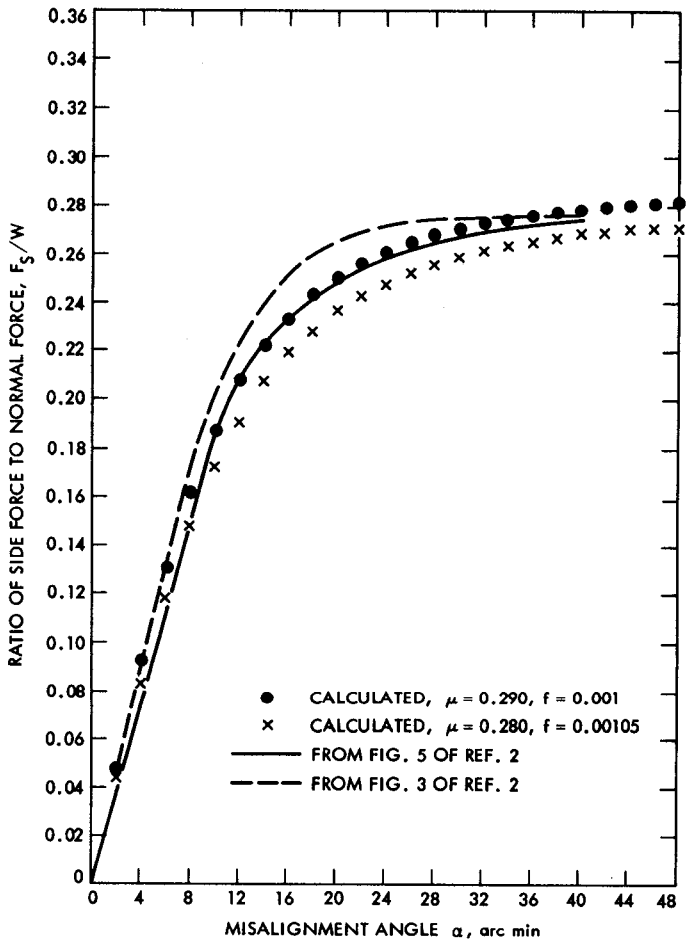

Fig. 3. Comparison of calculated lateral force coefficients with test results